Do these without a calculator.

① The region R is enclosed between the graph of the function $y = 2x - x^2$ and the x-axis. Sketch the region for $0 \le x \le 2$ and divide it into 4 subintervals. Sketch the rectangles used to compute the LRAM, RRAM, MRAM and find the area under the curve with each.

② Evaluate

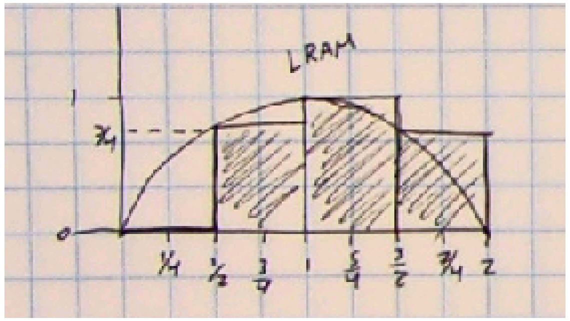
ⓐ $$\int_{-3}^{4} \frac{x^2-1}{x+1}\,dx$$

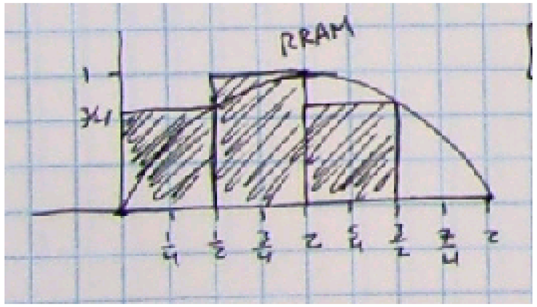
ⓑ $$\int_{a}^{b} 3t\,dt$$

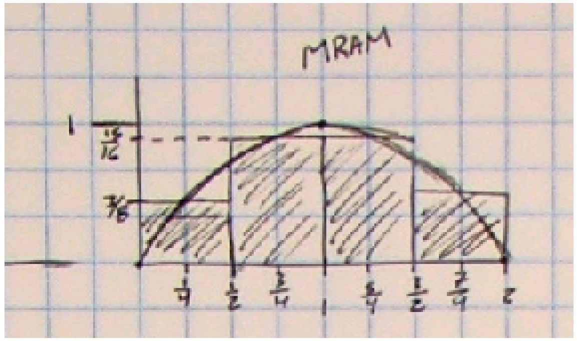
$0 < a < b$

ⓒ $$\int_{0}^{\frac{\sqrt{\pi}}{2}} -2x\sin(x^2)\,dx$$

$$\cos\left(\left(\frac{\sqrt{\pi}}{2}\right)^2\right) - \cos 0 = \frac{\sqrt{2}}{2} - 1$$





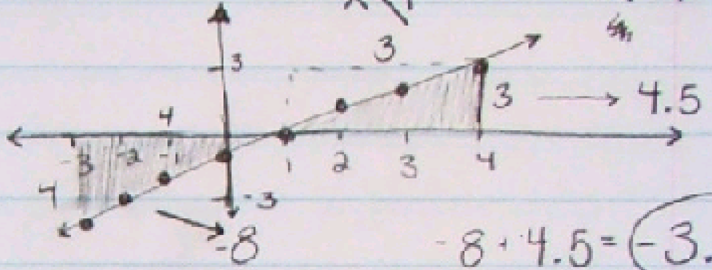


$.5(.75 + 1 + .75)$
1.25

1.25

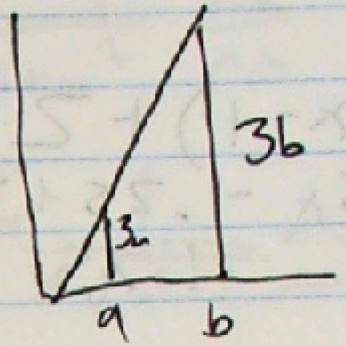
$.5(.4375 + .9375 + .9375 + .4375)$
1.375

2.a: $\int_{-3}^{4} \frac{x^2-1}{x+1}\,dx \longrightarrow \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} \longrightarrow x-1$



$-8 + 4.5 = -3.5$

b: $\int^{b}$



$\frac{1}{2}(3a+3b)(b-a)$

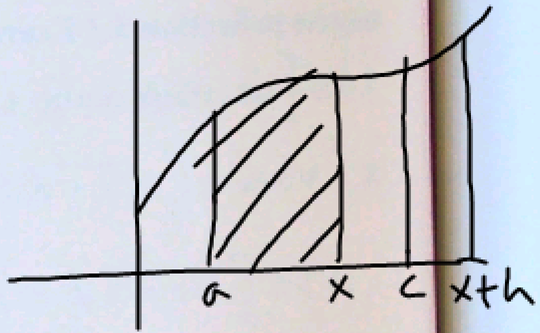
$\frac{1}{2}(\cancel{3ab} + 3b^2 - 3a^2 - \cancel{3ab})$

$\boxed{1.5b^2 - 1.5a^2}$

the
also
y ad-

Proof The geometric exploration at the end of the previous section contained the idea of the proof, but it glossed over the necessary limit arguments. Here we will be more precise.

Apply the definition of the derivative directly to the function F. That is,



$$\frac{dF}{dx} = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{\int_a^{x+h} f(t)\,dt - \int_a^{x} f(t)\,dt}{h}$$

$$= \lim_{h\to 0} \frac{\int_x^{x+h} f(t)\,dt}{h}$$

Rules for integrals, Section 5.3

$$= \lim_{h\to 0} \left[\frac{1}{h}\int_x^{x+h} f(t)\,dt\right].$$

The expression in brackets in the last line is the average value of f from x to $x+h$. We know from the Mean Value Theorem for Definite Integrals (Theorem 3, Section 5.3) that f, being continuous, takes on its average value at least once in the interval; that is,

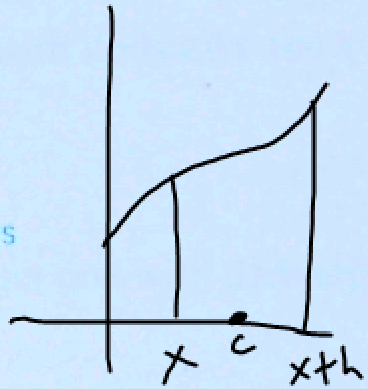
$$\frac{1}{h}\int_x^{x+h} f(t)\,dt = f(c) \quad \text{for some } c \text{ between } x \text{ and } x+h.$$

$$\frac{dF}{dx} = \lim_{h\to 0} \frac{1}{h} \int_x^{x+h} f(t)\, dt$$

$$= \lim_{h\to 0} f(c), \qquad \text{where } c \text{ lies between } x \text{ and } x + h.$$

What happens to c as h goes to zero? As $x + h$ gets closer to x, it carries c along with it like a bead on a wire, forcing c to approach x. Since f is continuous, this means that $f(c)$ approaches $f(x)$:

$$\lim_{h\to 0} f(c) = f(x).$$



Putting it all together,

$$\frac{dF}{dx} = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} \qquad \text{Definition of derivatives}$$

$$= \lim_{h\to 0} \frac{\int_x^{x+h} f(t)\, dt}{h} \qquad \text{Rules for integrals}$$

$$= \lim_{h\to 0} f(c) \qquad \text{for some } c \text{ between } x \text{ and } x + h.$$

$$= f(x). \qquad \text{Because } f \text{ is continuous}$$

This concludes the proof. ■

It is difficult to overestimate the power of the equation

$$\frac{d}{dx} \int^x f(t)\, dt = f(x). \qquad (1)$$

Find $\frac{dy}{dx}$

① $y = \int_2^x (3t + \cos t^2)\,dt$

$$\boxed{\frac{dy}{dx} = 3x + \cos x^2}$$

② $y = \int_0^{x^2} e^{t^2}\,dt$

$$\boxed{\frac{dy}{dx} = 2x\, e^{x^4}}$$

③ $y = \int_6^{x^2} \cot(3t)\,dt$

$$\boxed{\frac{dy}{dx} = 2x\cot(3x^2)}$$

④ $y = \int_x^6 \ln(1+t^2)\,dt$

$$\boxed{\frac{dy}{dx} = -\ln(1+x^2)}$$

⑤ $y = \int_{3x^2}^{10} \ln(2+p^2)\,dp$

$$\boxed{\frac{dy}{dx} = -6x\ln\left(2 + \underbrace{(3x^2)^2}_{9x^4}\right)}$$

⑥ Read example 4

EXAMPLE 4 Constructing a Function with a Given Derivative and Value

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$.

$\sqrt{\text{function}} = x$

$\sqrt{x^2} = x$

SOLUTION

The Fundamental Theorem makes it easy to construct a function with derivative tan x:

$$y = \int_3^x \tan t \, dt.$$

Since $y(3) = 0$, we have only to add 5 to this function to construct one with derivative tan x whose value at $x = 3$ is 5:

$$f(x) = \int_3^x \tan t \, dt + 5.$$

Now try Exercise 25.

Although the solution to the problem in Example 4 satisfies the two required conditions, you might question whether it is in a useful form. Not many years ago, this form might have posed a computation problem. Indeed, for such problems much effort has been expended over the centuries trying to find solutions that do not involve integrals. We will see some in Chapter 6, where we will learn (for example) how to write the solution in ... 4 as

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EXPLORATION 2 **The Effect of Changing *a* in $\int_a^x f(t)dt$**

The first part of the Fundamental Theorem of Calculus asserts that the derivative of $\int_a^x f(t)\,dt$ is $f(x)$, regardless of the value of a.

1. Graph NDER (NINT (x^2, x, 0, x)). $\longrightarrow$ x^2
2. Graph NDER (NINT (x^2, x, 5, x)). $\longrightarrow$ x^2
3. Without graphing, tell what the x-intercept of NINT (x^2, x, 0, x) is. Explain. x-int $x=0$
4. Without graphing, tell what the x-intercept of NINT (x^2, x, 5, x) is. Explain. x-int $x=5$
5. How does changing a affect the graph of $y = (d/dx)\int_a^x f(t)\,dt$? – No
6. How does changing a affect the graph of $y = \int_a^x f(t)\,dt$? – shift

Fundamental Theorem, Part 2

The second part of the Fundamental Theorem of Calculus shows how to evaluate definite integrals directly from antiderivatives.

THEOREM 4 (continued) **The Fundamental Theorem of Calculus, Part 2**

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

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Read rest 5.4

Do. #1-20 (3-5 more)

21-26 (3-5)